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Metamaterials Future of Cloaking

Theoretical Physics
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Metamaterials are one of the most exciting research areas in physics. The current knowledge and technology allows us to design, simulate and construct artificial materials that exhibit properties not found in any natural material. In the field of metamaterials, the freedom of design can allow scientists to construct materials that only existed in science fiction books. The work of the two pioneers, Victor Veselago and John Pendry, laid down the theoretical framework for the construction of the first negative refractive index materials. Such materials can manipulate electromagnetic radiation in new ways, giving rise to invisibility cloaks. Soon scientists went further and began pondering the possibility of designing metamaterials to cloak objects from thermal, acoustic, seismic and even matter waves. As a result, many interesting designs and properties have been discovered, although not all have yet been tested experimentally. The future of metamaterials is a fascinating one. Their applications will revolutionise our world. Scientists are constantly exploring new designs and applications.

Introduction

Metamaterials are the frontier of nanotechnology. These artificial materials can be specially designed to have many exotic properties. Negative refractive index metamaterials (NIMs) are just one class of materials in the general family of metamaterials. These are impossible to find in nature and have the potential to be used to fabricate invisibility cloaks. This review paper will focus mainly on the development, applications and future of NIMs. The two pioneers that ignited the curiosity driven research into metamaterials, NIMs in particular, are Viktor Veselago and Sir John Pendry. In 1967 Veselago published a paper in Russian entitled "The Electrodynamics of Substances with Simultaneously Negative Values of ϵ and μ". It was translated into English in 1968. He hypothesised that a material can possess a negative index of refraction if two parameters, electric permittivity (ϵ) and magnetic permeability (μ) are both negative. These two parameters are determined by the electrical and magnetic properties of substances. Figure 1 gives a visual summary of the ranges of values of ϵ and μ associated with different naturally occurring materials.

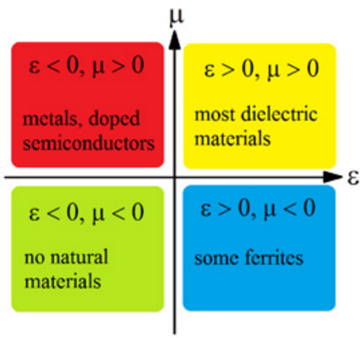

Figure 1: *Summary of the ranges of values of ϵ and μ associated with different naturally occurring materials.*

Veselago investigated the properties of materials with $\epsilon < 0$ and $\mu < 0$ in great detail. He provided the theoretical basis and the physical realisation of these materials.

The theory begins with an equation for the refractive index n

$$n^2 = \epsilon\mu \tag{1}$$

From equation 1, it may seem that the simultaneous change of sign of both ϵ and μ has no effect, but Veselago goes on further to show that setting $\epsilon < 0$ and $\mu < 0$ will make the material behave in ways only described in Science Fiction. To do this Veselago turns to equations where ϵ and μ appear independently. Now consider two of the famous Maxwell equations in matter, with no free charges and currents

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \tag{3}$$

where

$$\mathbf{B} = \mu\mathbf{H} \tag{4}$$

$$\mathbf{D} = \epsilon\mathbf{E} \tag{5}$$

Now apply these to the simplest case of a monochromatic plane wave, propagating through an isotropic, homogeneous medium, with electric and magnetic components given by

$$\mathbf{E}(\omega, \mathbf{k}) = \mathbf{E}_0 e^{(i\mathbf{k}\cdot\mathbf{r} - i\omega t)} \tag{6}$$

$$\mathbf{H}(\omega, \mathbf{k}) = \mathbf{H}_0 e^{(i\mathbf{k}\cdot\mathbf{r} - i\omega t)} \tag{7}$$

where ω is the angular frequency and $\mathbf{k}$ is the wave vector. The above equations yield

$$\mathbf{k} \times \mathbf{E} = \frac{\omega}{c}\mu\mathbf{H} \tag{8}$$

$$\mathbf{k} \times \mathbf{H} = -\frac{\omega}{c}\epsilon\mathbf{E} \tag{9}$$

From equations 8 and 9, it is clear that for $\epsilon > 0$ and $\mu > 0$ the three vector quantities $\mathbf{E}$, $\mathbf{H}$ and $\mathbf{k}$ form a right handed set, where for $\epsilon < 0$

and $\mu < 0$ the vectors form a left handed set.
Now consider the Poynting vector $\mathbf{S}$ defined as

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{H} \tag{10}$$

It follows from equation 10 that in right-handed materials, $\mathbf{S}$ and $\mathbf{k}$ are parallel and for left-handed materials $\mathbf{S}$ and $\mathbf{k}$ are antiparallel. Since $\mathbf{k}$ points in the direction of the phase velocity $\mathbf{v}_p$ of the wave, it follows that in left-handed materials the direction of $\mathbf{v}_p$ is opposite to the direction of $\mathbf{S}$, the energy flux.

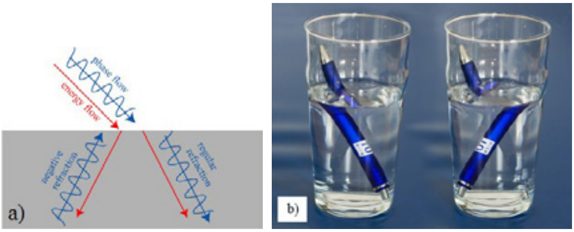

Figure 2: *a)Illustration of negative refraction at the interface of positive and negative refractive index materials. Used from (Skullsinthestars,2009).*
b)Diagram demonstrating refraction in left- and right-handed materials. Used from (Harshpaul 2011).

Figure 2(a) illustrates how waves behave at the interface of right-handed and left-handed media. Veselago goes on to derive the form of Snell's law for two media with opposite handedness, using the diagram shown in Figure 2(b)

$$\frac{sin\phi}{sin\psi} = \frac{n_2}{n_1} = \frac{p_2}{p_1}\sqrt{\frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}} \tag{11}$$

For a right handed medium, $p = 1$ and for a left handed medium $p = -1$, hence in the case where medium 1 is right handed and medium 2 is left handed, it follows that $\frac{p_2}{p_1} = -1$ and that medium 2, which is left handed with $\epsilon < 0$ and $\mu < 0$, has a negative index of refraction. (Veselago 1968)

Veselago predicted that NIMs will have many interesting properties such as reversed Doppler effect. From then on the challenge was to design the dream material. The first recipes were proposed by John Pendry (Pendry *et al.* 1996, Pendry *et al.* 1999). These were found to have many new properties, including negative refractive index (Shelby 2001). The applications of NIMs include perfect lenses (Pendry 2000) and invisibility cloaking (Pendry *et al.* 2006). The concepts of cloaking in electromagnetism can be applied to different areas. These materials can have many real-world applications that could transform our lives.(Kadic *et al.* 2013)

Engineering materials with $\epsilon < 0$ and $\mu < 0$

The flexibility of design in metamaterials allows for the construction of components, with a size smaller than the wavelength of radiation. Collectively, these produce artificial responses which results in a material with new properties. NIMs are constructed such that the artificial electric and magnetic responses yield negative permittivity and permeability. Pendry *et al.* (1996) described a structure that consists of infinitely long and very thin wires arranged in a simple cubic 3D lattice structure. In this structure the average electron density is reduced and hence the effective electron mass is greater, because of the self-inductance of the structure. The plasma frequency is depressed into the far infrared, which has been confirmed by simulations. The calculations show that the system can be described using the Drude-Lorentz model (Jackson 1999), hence the effective relative permittivity is given by

$$\epsilon_{r,eff}(\omega) = 1 - \frac{\omega_{p,eff}^2}{\omega(\omega + i\gamma_{eff})} \tag{12}$$

with the effective plasma frequency and effective damping factor given by

$$\omega_{p,eff}^2 = \frac{2\pi c^2}{a^2 \ln(\frac{a}{r})} \tag{13}$$

$$\gamma_{eff} \approx 0.1\omega_{p,eff} \tag{14}$$

where a is the lattice constant and r is the wire radius. From equation 12, it follows that $\omega < \omega_{p,eff}^2$ implies $\epsilon_{r,eff} < 0$.

Pendry *et al.* (1999) proposed structures with effective permeability that can be tuned to values not found in natural materials. One such structure is a split ring resonator (SRR), composed of two concentric split metallic rings,e.g. copper, with their openings positioned as shown in Figure 3.

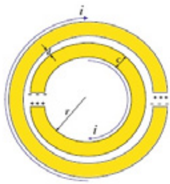

Figure 3: *The split ring resonator structure that defines c,d and r. Used from (wikipedia, 2012).*

The SRR is equivalent to an LC circuit. The magnetic flux passing through the SRR induces a strong current that generates a magnetic moment, in accordance with Faraday's law. If this magnetic response is sufficiently strong, it is possible to achieve a negative effective relative permeability $\mu_{r,eff}$. The expression for $\mu_{r,eff}$ is given by

$$\mu_{r,eff}(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\Gamma\omega} \tag{15}$$

where F is the filling ratio of the SRR

$$F = \frac{\pi r^2}{a^2} \tag{16}$$

ω_0 is the resonant frequency

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{3dc_0^2}{\pi^2 r^3}} \tag{17}$$

and Γ is the damping term

$$\Gamma = \frac{2}{r\sigma\mu_0} \tag{18}$$

where r, a and d are defined by Figure 3, c_0 is the speed of light in vacuum, σ is electrical conductivity of the metal and μ_0 permeability of free space (Pendry *et al.* 1999).

When the two kinds of meta-structures, one with $\epsilon_{r,eff} < 0$ and the other with $\mu_{r,eff} < 0$ in the same frequency range, are combined, it is possible to construct a material with $n_{eff} < 0$. Smith and colleagues constructed the first metamaterial composed of a periodic arrangement of long wires and SRRs as described above. This material exhibited negative ϵ and μ simultaneously for a narrow frequency range (Smith *et al.* 2000). Shelby, Smith and Schultz (2001) also constructed similar structures that exhibited a negative refractive index from 10.2 to 10.8 GHz. The image of their metamaterial structure and results are shown in Figure 4. Smith *et al.* (2002) reported on another material composed of SRRs and wires that exhibited a negative refractive index in the frequency region from 8.5 to 9.0 GHz.

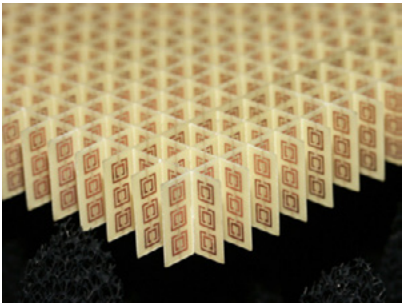

Figure 4: *Components of a negative refractive index metamaterial. These consist of copper SRRs and copper wire strips an a fibre glass circuit board material assembled in an interlocking lattice. Each unit cell has dimension of 5mm. Used from (Johnson 2008).*

After the achievements in the field of metamaterials and their possible applications, interest in this field grew rapidly. Now the goal was to manufacture materials that can couple to light at microwave and optical frequencies. Isotropic NIMs that can manipulate light at higher frequencies, with a broadband response and low energy losses required much smaller components and different material designs. Such materials could have many interesting applications such as perfect lensing, invisibility cloaking and many more. In this review paper, the focus is on cloaking using negative refractive index materials. (Pendry 2000, Pendry 2004, Pendry *et al.* 2006, Liu 2010).

Cloaking with Electromagnetic Negative Refractive Index Materials

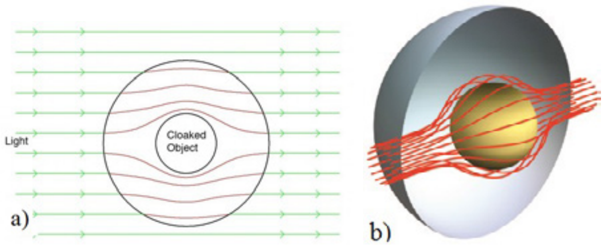

Figure 5: *a) Two dimensional cross section of the cloaking system showing the direction of rays inside and outside the metamaterial. Rays are diverted away from the cloaked circular area and emerge undisturbed from their original course, just as if it propagated through free space. Used from (Dodson 2013).*
b) A three dimensional view of the process described in a).
Used from (Sambles).

Pendry *et al.* (2006) proposed an approach for designing electromagnetic cloaks, based on transformation optics. Figure 5 shows how a spherical shell constructed from a negative refractive index metamaterial can alter the electromagnetic fields. The light rays come in and are deflected smoothly around the object and emerge in the same direction just as it would if it passed through empty space. No radiation enters or leaves the cloaked volume. Hence any object placed into the volume in the middle will appear invisible to the outside observer. This cloak does not depend on the shape or size of the object, as long as it fits inside the cloaked volume. The limitation in this case is that this cloaking can only be fully effective for one frequency.

This approach was then used by Schurig *et al.* (2006), who provided the first experimental proof of cloaking. The cloak was constructed from metamaterials that operated most effectively at 8.5 GHz. The material consists of copper SRRs in rectangular unit cells. These SRRs were specially designed to exhibit negative ϵ and μ. The metamaterial's overall structure is shown in Figure 6. The object being cloaked was a copper cylinder. The results of the simulations and measurements are presented in Figure 7.

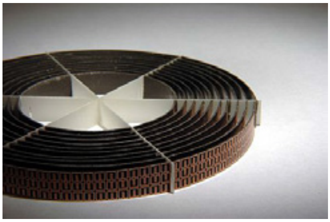

Figure 6: *The structure of the two dimensional cylindrical cloaking structure. The shape of SRRs varies gradually in the structure at every cylinder. Used from (Gbur 2011).*

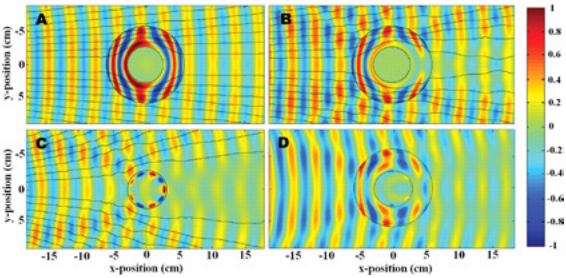

Figure 7: *Diagrams of steady-state electric field pattern with stream lines (a - c) indicating the direction of the Poynting vector. The cloak lies in the region between the two concentric circles. Inside the smaller circle lies a copper cylinder. The right-hand scale shows the instantaneous intensity. a)Simulation of the cloak with exact material properties. b)Simulation of the cloak with reduced material properties. c)Experimental measurement of the copper cylinder. d)Experimental measurement of the cloaked cylinder.Used from (Schurig 2006).*

While this design gives excellent results, it is only valid for a very narrow frequency band. Also, this design is not practical for engineering of optical metamaterials.

Valentine *et al.*(2009) reported on the first demonstration of optical cloaking using a carpet cloak. This material consists of isotropic dielectric materials, which allows it to operate for a broad frequency band and produce low-loss invisibility for a wavelength range of 1400-1800 nm. The carpet cloak is formed by etching holes in a silicon slab using a focused ion beam (FIB). The carpet cloak was used to transform a mirror with a bump, into a virtually flat mirror. Any object placed behind the bump will be invisible because the reflected beam will appear to have bounced off a flat mirror surface. The structure of the carpet cloak is show in Figure 8. The results show that a beam reflected off a perfectly flat surface has a gaussian intensity profile. The light hitting the mirror with a bump scatters and the profile has three lobes. The light reflected off the cloaked bumped surface has a profile very similar to the gaussian profile of a perfect mirror.

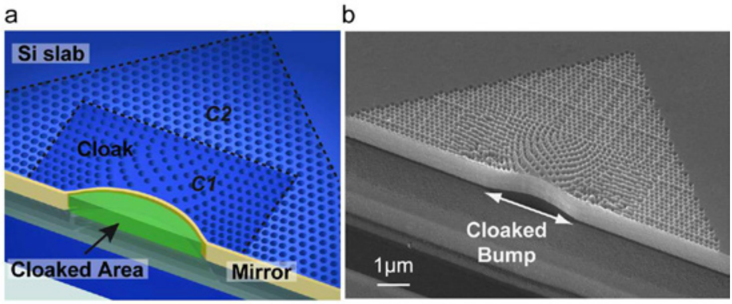

Figure 8: *a) Diagram of the structure of the carpet cloak. The triangular region (C2) has a uniform hole pattern and serves as a background medium with constant effective refractive index. The rectangular region C1 has a variable index profile. b) SEM image of the structure of the carpet cloak. Used from (Yaris 2009).*

The work on creating better metamaterials and cloaks, operating at even higher optical frequencies is on-going. The results so far have been very promising. In the future we can expect the making of bulk metamaterials on much larger scales. We will not only have invisibility cloaks, but also perfect lenses, perfectly absorbing materials, etc. These materials and many more will transform the world.

Beyond Electromagnetic Cloaking

The concepts of cloaking in electromagnetism can be applied to other areas of physics to give rise to thermodynamic, acoustic, elastodynamic and, in principle, matter-wave cloaking metamaterials. These materials could have scientific and real world applications and hence it is an important topic of research. Principles analogous to those of transformation optics can be used to produce cloaking devices that will manipulate waves in matter, e.g. thermal, acoustic, seismic, etc. Since matter itself is a wave, theory suggests that in principle, matter cloaking is possible. This last section will briefly outline the work done on the theory and design of such metamaterial cloaks and their possible applications in that future.(Kadic 2013)

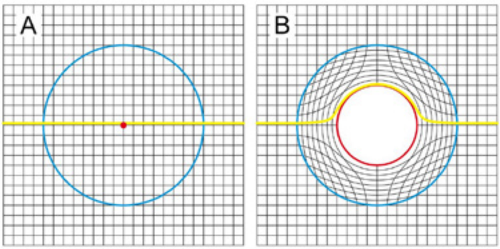

Figure 9: *Principle of cloaking. a)A simple square grid. b)After applying an appropriate coordinate transformation, a distorted square grid with a hole in the middle is obtained. Used from (Leonhardt 2011).*

Thermal, acoustic and seismic cloaking are based around a similar principle. In thermal cloaking, the grids in Figures 9 (a) and 9 (b) are composed of thermal and electrical resistors. Conductivity locally varies radially and azimuthally, however a measurement of heat flux taken by an observer outside of the region marked with the blue circle would show that the two constructions indistinguishable. Hence an object placed inside the region marked with the red circle can be cloaked. Using this principle Schittny *et al.* (2013) constructed and characterised the first thermal cloak. Certainly such cloaks could have many applications in technology and industry. In acoustic cloaking, the grids in Figures 9 (a) and 9 (b) are composed of springs. The idea is then exactly the same as with the thermal cloaks, just now the springs represent the bulk modulus. Once again the bulk modulus varies radially and azimuthally, however outside the blue circle line the two constructions are indistinguishable because phase velocity of the sound wave measured outside the blue circle will be the same for both constructions. The design for a 2D acoustic cloak was first proposed by Cummer & Schurig (2007), but 3D model was also proposed (Chen & Chan 2007). Acoustic metamaterials could be used to create a seismic shadow zone. In this scheme, the seismic waves could be converted to sound and heat (Kim & Das 2013). A seismic cloak constructed from isotropic heterogeneous thin plates, that could deflect seismic waves around buildings has also been investigated(Farhat *et al.* 2009). The obvious application of seismic cloaks is to shield buildings from earthquakes, which would save peoples' lives and protect buildings from damage. Another possible application could be to deflect shock waves in car accidents, which could also save people from serious damage and death. One type of cloaking that has not yet made a definite transition from science fiction into science fact is the quantum-mechanical matter wave cloaking. If an object is cloaked by, say an optical cloak, one way of finding it would be to throw an object in its direction and watch how it scatters. In principle, once could cloak themselves from the projectile by generating a special potential-energy distribution. However such a cloak could only work if the projectile had a particular incidence energy(Kadic *et al.* 2013). However Greenleaf *et al.*(2008) proposed a way of approximate quantum cloaking using a potential. If a potential is surrounded by an approximate cloak, it would be unaltered and undetectable to particles. At some energies, matter waves could become trapped in the cloak and this could have applications in beam switches and magnetically tunable ion traps.

Conclusion

Since Veselago gave his mathematical description of negative refractive index, the scientific world had to wait almost thirty years for Pendry and others to come along and turn metamaterials into the exciting area of research it is today. At the beginning of the new millennium, the theoretical work of Veselago and Pendry was verified experimentally by Smith and others. Since then many more metamaterials with exotic properties were designed and some were constructed. Invisibility cloaks made the transition from science fiction to science fact. Possible applications in the real world will transform our society and technology. Electromagnetic cloaks can smoothly guide radiation around objects. Acoustic and seismic cloaks could protect houses from the damages of the powerful shock waves produced by earthquakes. Thermal cloaks could shield objects from heat. The metamaterial world is just waiting to be further explored.

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