Inside the "Upside Down": Estimating Ireland's Output Gap

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Abstract: This paper attempts to identify estimates of Ireland's output gap that are relevant for fiscal policy. In contrast to standard approaches, we focus on measures of domestic economic activity, given its relatively more tax-rich nature. We examine and test various methods based on univariate/multivariate filters and principal components analysis, comparing our estimates with those of the EU Commonly Agreed Methodology. We find that our results are stable; are less complex in structure; are able to explain price and wage inflation; and, most importantly, yield estimates that are more plausible for Ireland.

I INTRODUCTION

This paper attempts to identify estimates of the output gap for the Irish economy, particularly those that are relevant for designing an appropriate fiscal policy. The output gap is a summary measure of the difference between the economy's actual level of output and the level of output that would be expected if the economy were at its most efficient – that is, at full capacity.

Determining the current budgetary stance and its sustainability requires an understanding of the cyclical position of the economy and its potential growth rate. An economy operating above its potential (i.e. where the output gap is positive) would be expected to show stronger government balances than one in steady state. Revenues would be expected to be higher, and cyclical expenditure on areas like unemployment benefits would be expected to be lower. It is also important to have

a medium-term view as to economic growth prospects in the context of expenditure planning and debt sustainability assessments; something that estimates of potential output can help to determine.

A number of challenges face us; particularly Ireland's small, open nature, and the presence of large foreign-owned multinational enterprises. Recognising these challenges, we prioritise measures that focus on domestic activity, an approach warranted given its relatively more tax-rich nature. By comparison, standard approaches, such as the EU Commonly Agreed Methodology (Havik *et al.*, 2014), focus on GDP.

Given the significant uncertainty surrounding output gap estimates, it would be unwise to focus on any single approach. Individual approaches are likely to be driven by specific inputs or assumptions that can fail to deliver answers that are consistently plausible. We therefore adopt a "suite of models approach" that emphasises the use of a range of alternative estimation techniques rather than relying on any single approach. This can be helpful when faced with uncertainty, and is shown to outperform single models in a forecasting context.

II RELEVANT LITERATURE

2.1 Statistical Filters

Statistical tools such as the Hodrick-Prescott filter (Hodrick and Prescott, 1981) and the Kalman filter can be used to extract a smoothed trend from an output series. If the trend approximates the path of potential output, the output gap can be measured as the gap between trend and actual output levels.

There are three common criticisms of these approaches. First, they incorporate little, if any, theoretical foundation and draw on limited economic information. As such, they are said to represent purely statistical approaches. Second, some of the dynamics of trends produced may not be sensible for economic variables. The dynamics assumed in a HP filter may be an unreasonable representation of the underlying data-generating process and structural breaks may be smoothed to an unreasonable degree (Hamilton, 2017; Ódor and Jurašeková Kucserová, 2014). Third, the so-called "end-point problem" can, with some filters, result in estimates that are highly biased at the ends of the sample. This occurs in a fashion that is typically procyclical (i.e. the smoothed series tends to be close to the observed data at the beginning and end of the estimation sample). In practice, sufficient new observations are required before a satisfactory decomposition of data into its trend and cycle components may be achieved. This problem is often offset – though not eliminated – by extending historical data with forecast observations.[1]

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[1] Mohr (2005) notes that this bias can occur even "if the forecast itself is unbiased and the forecast error is a random white noise process". This reflects the fact that the implied errors in the computation of the trend are unlikely to share the more desirable features (white noise, and random) of the forecasts errors. The filter model of course differs from the model that underlies the forecast.

There have been efforts to introduce some additional information to statistical filters, while drawing on economic theory. Statistical filters can be extended to include additional economic variables that contain some information about the cycle. The latter are generally termed "multivariate filters". A seminal work in this area is presented in Borio *et al.* (2013; 2014) where additional variables are incorporated in a multivariate filter setting that also employs Bayesian techniques for the US, UK and Spain. A mix of financial variables (credit growth, real interest rates, and housing prices) and other economic variables (capacity utilisation measures, inflation, and unemployment rates) are considered in order to better filter out underlying trends from the cycle. The approach is also availed of in work such as Darvas and Simon (2015) for 12 EU economies and five non-EU economies; in Alberola *et al.* (2014) for Spain; and in Ódor and Jurašeková Kucserová (2014) for Slovakia.

Other variables incorporated in the multivariate setting include the current account, financial and asset/commodity prices. Incorporating the current account is a means of identifying the absorption cycle (Bénétrix and Lane, 2015). Dobrescu and Salman (2011) and Lendvai *et al.* (2011) emphasise the role of the current account deficit in augmenting fiscal cycles, for example. Bornhorst *et al.* (2011) argue for consideration of asset and commodity price cycles (e.g. oil prices in heavily resource-dependent countries or, alternatively, real estate and equity prices). Though multivariate filters help to address the problem of a lack of theoretical foundations, they may still have a number of conceptual weaknesses. A key issue is whether financial cycles can be identified in real time (Blagrave *et al.*, 2015).

2.2 Production Functions

Estimates of potential may also be obtained based on assumptions regarding the potential level of factor inputs like capital and labour along with Total Factor Productivity – the efficiency with which factor inputs are used to produce output. This approach is currently agreed by EU Member States and used by the European Commission (EC) in assessing compliance with legislated fiscal rules (Havik *et al.*, 2014); hereafter, we refer to this agreed EU approach as the Commonly Agreed Methodology (CAM). Other organisations such as the OECD also currently employ variants of the production function approach (Turner *et al.*, 2016; Johansson *et al.*, 2013; Giorno *et al.*, 1995).

There are a number of weaknesses to production function approaches such as the CAM. These particularly relate to how labour, capital and total factor productivity are incorporated.

Applicability to open economies: The production function approach can disregard certain behaviours of open economies where excess demand may be absorbed by the trade balance or, more broadly, by the current account balance. This phenomenon is consistent with the absorption cycle (Lendvai *et al.*, 2011). Sharp deteriorations in these balances were evident in Ireland, Greece, Latvia, and

Spain prior to the great recession (Darvas, 2013). In this vein, Darvas (2015) shows that the size of revisions to CAM-based output gap estimates are correlated with the variability of the current account balance, suggesting that important information is not utilised in the EC and IMF output gap estimates.

Capital: Approaches such as the CAM see growth in the level of the actual net capital stock as driving the capital contribution to potential output. However, identifying sustainable levels of output linked to capital may be complicated. First, there are significant issues involved in measuring the capital stock accurately (OECD, 2001) with major challenges posed by the openness of capital (Fratzscher and Bussiere, 2004; Obstfeld, 1985). Unsustainable developments, such as asset price bubbles in the housing sector, can also distort capital contributions to potential output. For example, investments into housing may boost capital levels, thus inflating potential output as measured. However, the actual effects on an economy's potential might best be considered unsustainable over the long term.

Labour: The contribution from labour inputs in the production function approach is often centred on the identification of the NAWRU (Non-Accelerating Wage Rate of Unemployment); i.e. the level of unemployment that keeps inflation constant. The CAM production function obtains the implied trend unemployment rate based on a version of an accelerationist Phillips curve. Combining this with trend labour force levels gives trend employment levels, which, together with trend average hours worked, gives the total potential level of factor inputs from the labour side (i.e. trend total hours worked).

The estimation of the NAWRU has been a focal point for recent criticism of the production function approach employed under the CAM (e.g. Fioramanti, 2016; Darvas and Simon, 2015). Of utmost concern is the extent to which the estimates can tend to track actual unemployment for some economies. Rather than identifying a persistent trend unemployment rate, the NAWRU appears to more closely approximate the actual unemployment rate (87 per cent of the observations for Ireland since 1980 fall between +/– 2 percentage points of the actual rate). This tendency occurs whenever actual unemployment experiences sharp swings, even in the absence of developments that might explain rapid shifts in structural unemployment. For example, the NAWRU estimates for Ireland in the late 2000s onwards rise and fall sharply in step with actual unemployment rates, despite the absence of major labour market reforms that might explain such drastic changes in the NAWRU.

Total Factor Productivity (TFP): The CAM production function approach identifies trend TFP as the smoothed time series of the Solow residual. The latter is obtained from actual real GDP data based on assumed output elasticities of capital and labour inputs.[2] A key challenge when attempting to identify trend TFP is the instability of the estimates toward the end of the sample period. The CAM addresses

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[2] The CAM assumes that these are approximately ⅓ and ⅔, respectively.

this problem with two solutions: (1) forecasts are incorporated to overcome the end-point problem by effectively extending out the sample used; and (2) a Kalman filter is used rather than a HP filter as it is understood to suffer relatively less from end-point bias. This is thanks in large part to its ability to incorporate economic information from elsewhere, unlike the typical HP filter approach. The CAM draws on a time series for capacity utilisation to help the Kalman filter to identify estimates, given that this may have additional information about the TFP cycle.

Notwithstanding these useful innovations, problems remain. First, bias in the forecast observations can prompt changes in the trend series produced (for historical and forecast years) so that the addition of forecasts may be unhelpful (or worse, misleading). Second, regardless of any bias in the forecasts, there is a tendency for estimates to converge on forecasts. Rather than overcoming end-point bias, therefore, the effect of including forecasts may simply be to push end-point bias out to later periods in the forecast horizon. Third, augmenting the series with information from the capacity utilisation series may be helpful provided that the series offers some useful informational content. However, such surveys have their limitations: data are not available for forecast periods; the surveys are typically limited to manufacturing sectors; and the quality of information provided is often poor.[3]

2.3 Cyclical Indicators

Another useful approach for estimating the output gap is to account for a wide range of indicators of the cyclical position of the economy. Such an approach is currently used by agencies like the Office for Budget Responsibility (OBR) in the UK (Murray, 2014; Pybus, 2011) and it is also shown to be usefully applied to measure the Euro Area aggregate business cycle (Altissimo *et al.*, 2001). Indicators are chosen so as to reflect cyclical factors and may include a variety of survey measures of spare capacity and recruitment difficulties as well as measures of earnings growth. Prior information on sectoral shares or statistical techniques may be used to derive weights for each of the indicators in order to produce an overall measure akin to an output gap. One approach to the estimation process, based on the method of principal components, involves assigning weights to each of the indicators employed so that the derived output gap series explains as much of the variability of the data as possible.

The Cyclical Indicators approach has the advantage of being able to overcome the usual reliance on output estimates that can be frequently prone to substantial revisions. This is particularly an issue for Irish GDP data, which are shown to have among the largest revisions in the OECD (Casey and Smyth, 2016). In addition, because the indicators chosen to underpin the output gap estimates avail of data

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[3] For example, it can be unclear who responds to the survey and how exactly they interpret its questions (Bauer and Deily, 1988). For Ireland, limitations are especially pronounced, given that the series itself has been discontinued, with the last observation collected in 2008 (Clancy, 2013).

that tend to be unrevised, this means that real-time estimates are unlikely to differ very much from those which might be estimated at another point in time (Murray, 2014). A difficulty posed by the approach is that cyclical indicators identified need to be combined and weighted to produce an aggregate output gap. This is not a straightforward process. Indicators are typically very different and require transformations to produce comparable series. Furthermore, it is not necessarily obvious what an appropriate weight might be for each given series. More fundamentally, the indicators selected themselves might not provide a sufficiently comprehensive picture of cyclical developments.

2.4 Suite of Models Approach

We adopt a "suite of models approach" as an attempt to overcome the uncertainties faced. The diversification afforded by this approach is one way of reinforcing the robustness of the estimates produced. There are obvious practical limits to the informational content of any single model, while the relevance of any single model paradigm may also vary over time (e.g. following the recent financial crisis). In addition, there may be a number of specific factors that we may be interested in, which individual models may fail to address if relied upon in isolation. It is generally accepted that diversification can lead to more robust forecasts/estimates in the face of uncertainty. Empirical work by Bates and Granger (1969) and Stock and Watson (1999) shows that the suite of models approach tends to outperform single models in a forecasting sense. The approach is usefully applied for short-term forecasting in the Irish case in Conroy and Casey (2017). A similar approach is advocated by the OBR (2011) for the UK output gap given the assertion that "it would be unwise to base an assessment of economic prospects on any single approach alone" (p.6). With output gap estimates, as with forecasts, there are obvious practical limits as to how informative any single approach can be. By having a range of models that incorporate different information about the cycle, it is hoped that key developments in relation to the cyclical position of the economy will be captured.

III ISSUES RELEVANT FOR IRELAND'S OUTPUT GAP

3.1 Features of the Irish Economy

Producing output gap estimates for an economy always presents difficulties because of its unobservable nature, as well as other common problems. These problems are well documented in the literature (Section II). However, estimating the output gap for the Irish economy poses considerable difficulties: (i) the openness of the economy, (ii) its small size, and (iii) the presence of large foreign-owned multinational enterprises. In particular, these factors mean that large changes in activity can result from a small set of large exporting enterprises. Owing partly to

their strong integration in global supply chains, such enterprises are capable of varying their production substantially with little impact on domestic factor inputs or domestic capacity utilisation.

Ireland's integration with the global economy is among the highest observed internationally. In terms of the scale of traded activity relative to the size of the economy, Ireland ranks second highest in the OECD, both in terms of GNP and GDP. Based on 2014 data, Ireland is in the bottom 20 per cent of OECD countries in terms of size (based on working age population). Looking at size on the basis of levels of GDP in US Dollars, Ireland ranks 24th among 35 OECD countries, while using GNP just for Ireland, it ranks 27th.

Another important feature of the Irish economy is the role of highly productive sectors where foreign-owned multinational enterprises dominate. On the one hand, an estimated 2.2 per cent of enterprises in the business economy in Ireland are foreign-owned (2012 data), yet these enterprises account for an estimated 58.4 per cent of total GVA. On the other hand, resident-owned enterprises account for 97.8 per cent of such enterprises, but less than half (41.6 per cent) of total GVA. This dichotomy might lead to a characterisation of Ireland as having a two-speed economy. One part of the economy is marked by strong growth and high productivity. These activities are concentrated in "modern" exporting sectors that have comparatively stronger integration in global value chains. The other part of the economy is marked by weaker growth in relatively less productive and less globally integrated "traditional" sectors of the economy.[4]

3.2 Implications for Output Gap Estimates

The issues described above pose significant challenges and it is unlikely that any single solution will adequately address all of these. One useful solution for many of the distortions caused by the foreign-owned multinational-dominated sectors is to focus on measures of output that distinguish between these sectors and the rest of the economy. The motivation for this kind of approach is even greater if we are interested in an output gap that is relevant for Irish fiscal policy. This is due to the evidence that the relationship between tax receipts and aggregate activity (e.g. GDP and GNP) is weaker than that for domestic measures. By contrast, the relationship between revenues and output from the foreign-owned multinational-dominated sectors tends to be insignificant (IMF, 2015).

The Table 1 regressions show the association between total general government revenue and different measures of nominal aggregate output. The CSO provides a split of Gross Value Added (GVA) into that of sectors that are dominated by foreign-owned multinational enterprises (GVA of MNEs) and the rest of the economy

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[4] Considerable efforts have been made to strip out distortions related to sectors that provide little in the way of changes to "real" variables like employment or wages.. Most recently, an alternative indicator of Irish economic activity: "modified GNI" or "GNI*", has been introduced (CSO, 2017).

(Domestic GVA). Regressions (3) and (4) show that changes in GNI* and Domestic GVA explain more of the variation in revenue growth than changes in headline measures such as GDP (1) and GNP (2). By contrast, regression (5) indicates that changes in the gross value added of sectors dominated by MNEs are estimated to have no statistically significant impact on revenues.

Table 1: Output and General Government Revenue
Dependent Variable ΔRevenue$_t$/Revenue$_{t-1}$ (Sample: 1990-2016)

Regression	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*
ΔGDP$_t$/GDP$_{t-1}$	0.6888***				
	(0.1898)				
ΔGNP$_t$/GNP$_{t-1}$		0.9665***			
		(0.2064)			
ΔGNI*$_t$/GNI*$_{t-1}$			0.8439***		
			(0.1026)		
ΔDomestic GVA$_t$ / Domestic GVA$_{t-1}$				1.6280***	
				(0.2290)	
ΔGVA of MNEs$_t$ / GVA of MNEs$_{t-1}$					0.0244
					(0.0327)
Constant	2.4273	1.4678	0.4217	0.0065	5.8601
	(1.4941)	(1.3974)	(0.9789)	(1.1533)	(1.4162)
Observations	27	27	27	27	27
R-squared	0.34	0.47	0.73	0.67	0.02
Root Mean Squared Error	5.18	4.67	3.32	3.68	6.33

Sources: CSO; author's calculations.
Notes: Robust standard errors in parentheses (*** $p<0.01$; **$p<0.05$; *$p<0.1$). Revenue refers to total General Government Revenue. Domestic GVA is total GVA less sectors dominated by foreign-owned multinational enterprises (GVA of MNEs). GNI* is an aggregate that is designed to more accurately capture national income of Irish residents compared to GDP, given that GDP is prone to distortions from foreign-owned multinational enterprises. GNI* differs from actual GNI in that it excludes (i) the depreciation of foreign-owned, but Irish-resident, capital assets (specifically, intellectual property and aircraft leasing assets) and (ii) the undistributed profits of firms that have re-domiciled to Ireland. For years where GNI* data are unavailable (1990-1994), we extend the GNI* series using the unadjusted GNI series growth rates.

Distinguishing between sectors dominated by foreign-owned multinational enterprises and other ("domestic") sectors enable us to focus on producing output gap estimates that are more relevant for fiscal policy. It also helps us to overcome distortions caused by large shifts in activity attributable to multinational enterprises. However, the distinction between domestic and non-domestic may not be perfect.

Another alternative approach we explore is to use additional economic indicators that might provide information more closely linked to domestic cyclical developments (in a Cyclical Indicators approach).

IV METHODOLOGY AND DATA

This section details the methodologies and data we use to estimate Ireland's output gap.

4.1 Methodology

4.1.1 Statistical Filters

A range of statistical filters may be applied to individual measures of economic activity (e.g. real GDP) to obtain a smoothed trend series, which may then be considered the economy's level of potential output.[5] For this paper, we consider two of the most common tools, the Hodrick-Prescott (HP) filter and the Kalman filter. The output gap is then defined as the difference between trend and actual output levels (expressed as a percentage of the trend level).

The HP filter is a simple smoothing method which obtains time-varying trend estimates by minimising:

$$\Sigma_{t=1}^{N} \left[(Y_t - Y_t^*)^2 + \lambda(\Delta Y_t^* - \Delta Y_{t-1}^*) \right] \tag{1}$$

where Y_t is the output variable of interest (such as real GDP) and Y_t^* is the unobserved trend estimate that we wish to identify. The method (i) minimises the sum of the squared deviations between output and its trend $(Y_t - Y_t^*)$, while (ii) minimising the change in the trend growth rate from one period to another. The lambda (λ) parameter allows some flexibility in relation to the smoothness of the extracted trend with the potential output estimates approaching a linear trend for larger values. It is at the discretion of the user how smooth this parameter is set to be, but typically 100 or 6.25 are assumed when identifying trend estimates for the business cycle with annual data.[6] We explore both in the case of the output measures we consider.

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[5] See Cerra and Saxena (2000) for an overview.

[6] The choice of smoothing parameter is widely debated and often tends towards what are considered "de facto industry standards" (Maravell and del Rio, 2001). Hodrick and Prescott (1997) themselves suggested a lambda of 100 for annual data and 1600 for quarterly data. Ravn and Uhlig (2002) recommend either (i) varying lambda according to the frequency of the data such that it varies by the fourth power of the frequency observation ratio (e.g. 1600 for quarterly data and $1600/4^4 = 6.25$ for annual data) or (ii) a time domain approach that determines lambda using the ratio of the variance of cyclical components to the variance of the second difference of the trend component so as to allow for idiosyncrasies in the data. In practice estimates will be sensitive to the choice of smoothing parameter. For annual data, the standard Hodrick and Prescott (1997) lambda of 100 implies a cycle of 19.8 years, while the lambda of 6.25 is consistent with a ten-year cycle.

The Kalman Filter is a variant of state-space models – a general class of linear time series models that combine observable variables (X_t) and unobservable (S_t) variables. They can be described by two equations.

The first "state" or "transition" Equation (2) describes how a set of unobservable state variables, S_t, evolve over time. The second "measurement" or "signal" Equation (3) relates a set of observable signal variables, X_t, to the unobservable state variables:

$$S_t = FS_{t-1} + u_t \qquad (2)$$

$$X_t = HS_t + v_t \qquad (3)$$

The error terms u_t and v_t are serially independent and may include errors that are normally-distributed or can rely on other distributional assumptions.

The intuition behind state-space models is straightforward. Not being able to observe S_t, we make do with an observable, unbiased estimate based on information available up to time t−1. This estimate, called $S_{t|t-1}$ (the estimate of S_t conditioned on information from preceding periods) is equivalent to the left-hand side of Equation (2), and its errors can be assumed to be, for example, normally distributed. Substituting this into Equation (3), we can describe the observed variables as:

$$X_t = HS_{t|t-1} + v_t + H(S_t - S_{t|t-1}) \qquad (4)$$

Since $S_{t|t-1}$ is observable and we assume that the unobservable elements (v_t and $(S_t - S_{t|t-1})$) are normally distributed, the model can be estimated via maximum-likelihood methods. Given initial estimates of the first-period unobservable state $S_{1|0}$, the combined likelihood for all subsequent data observed is the product of all the period-by-period likelihoods.

The iterative method used to produce the unbiased estimate of our unobservable variable based on information available up to time t − 1 (i.e. $S_{t|t-1}$) can be understood as follows. First, given the observed signal variables and some initial assumptions about state mean and variance values, the Kalman filter calculates one-step-ahead estimates of state values and variances. This gives an initial projection of the state variable. Second, the observable data for the next period is used to update the projections from step 1, giving more weight to components with lower variances. In step 2, the error covariance is also corrected with the same weight as the prior estimate of the state variable (Harvey, 1989).

As a starting point, we use the following state-space system representation to model the stochastic process for output:

$$y_t = OG_t + y_t^* \qquad (5)$$

$$OG_t = \beta_1 OG_{t-1} + \beta_2 x_{t-k} + \omega_t \tag{6}$$

$$y_t^* = y_{t-1}^* + \varepsilon_t \tag{7}$$

where y_t is the log-level of actual output; y_t^* is the log-level of potential output; and OG_t is the output gap all as measured for the current year t. The variable x_{t-k} is an optional vector of economic variables that can enter the equation in the multivariate setting. For our core univariate model, this is set as equal to zero.

The system above assumes the following relationships. First, our actual output variable (e.g. real GDP) is set as equal to potential output plus the output gap. Second, the output gap is modelled as a stochastic process that evolves with an autoregressive component, and with white noise and normally distributed shocks given by ω_t. The shocks in the output gap equation can be thought of as cyclical or transient demand shocks. Third, potential output is assumed to evolve with an autoregressive component also, and, again, with white noise and normally distributed shocks ε_t. The shocks in the latter potential output equation may be deemed as level shocks to potential output.[7]

In an extension of the above, we include a time-varying drift term δ_t (which, itself, is a random walk also) to our third Equation (7) so that we have:

$$y_t^* = y_{t-1}^* + \delta_t + \varepsilon_t \tag{8}$$

To estimate, first we specify that the parameters are normally distributed and initialise the model with the prior that variances are given by a simple HP filter of historical data. Second, we employ the Kalman filter to estimate the likelihood of the system. Thus, we maximise the posterior density function with respect to our parameters. This can be viewed as a conventional Bayesian approach to estimating parameter values and the variances of shock terms.[8]

4.1.2 Multivariate Statistical Filters

A useful extension of the state-space representation outlined above is to include a number of additional exogenous and observable economic variables. While the equations in the previous section form the core of our state-space system representation, we can include additional signal variables to arrive at a multivariate

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[7] Another possibility to explore in this framework would be the inclusion of a fourth equation capturing trend growth G_t and the possibility of shocks to this. As in Blagrave *et al.* (2015), this could be modelled in the fashion: $G_t = \theta G^{SS} + (1-\theta) G_{t-1} + \varepsilon_t^G$. This approach would see trend growth G_t as being subject to trend growth rate shocks ε_t^G, the impact of which can fade over time according to the persistence parameter θ (with smaller values giving more persistent trend growth shocks). The trend (or potential) output equation would be modified to include this trend growth term G_t so that we have: $y_t^* = y_{t-1}^* + G_t + \varepsilon_t$.

[8] As noted in McGrayne (2012), though Kalman vehemently denied that Bayes' theorem had anything to do with his invention, Masanao Aoki proved mathematically in 1967 that it can be derived directly from Baye's rule. For a general discussion of how the Kalman filter is used to obtain estimates of the unobservable variables, see Hamilton (1994).

representation. The idea is to complement our output aggregates with additional economic information about the cycle.

In this context, we examine a number of possible indicators to include in the vector x_{t-k}. We examine both contemporaneous interactions and one-year lags. As we introduce the economic indicators assessed via the output gap equation, we are effectively only allowing them to have an indirect impact on potential output. This approach means that such variables are assumed to only contain information about the cyclical or transitory components of output. Of course, many shocks (e.g. financial sector crises) could be argued to have permanent negative impacts on potential output. This possibility is allowed for in an indirect sense under our specification of the output gap equation. As in Borio *et al.* (2013), this constrains potential output to be proportional to actual output, so that any permanent effects, if relevant, will ultimately be reflected in potential output also.

Following the literature, we examine the following economic variables for inclusion. As in Borio *et al.* (2013), we examine a measure of private sector credit growth, residential property prices and the real interest rate. These are intended to help capture the interactions between financing constraints, collateral values and wealth effects. We also consider the real effective exchange rate and a modified version of the current account balance.[9] Several transformations of each of these variables are examined: mean adjustments on the basis of simple arithmetic averages; mean adjustments on the basis of Cesàro averaging; and gaps produced under a univariate application of the HP filter. Following Borio *et al.* (2013), we estimate the inclusion of each variable, one at a time. This sequential approach allows us to assess the effect that each variable introduced in isolation has on our output gap estimates.

4.1.3 *Cyclical Indicators*
The cyclical indicators approach is another useful approach for identifying the cycle. It exploits information from a range of variables that might be expected to reflect cyclical developments and is typically applied to survey indicators rather than aggregates like GDP. Measures include various high frequency indicators such as those provided in surveys of consumer and business conditions, labour market indicators and financial indicators. To combine the cyclical indicators used, we use Principal Components Analysis (PCA), a statistical technique that attempts to identify the common determinant of a number of variables and to account for as much variability in the data as possible. It assigns weights to each of the variables according to the underlying properties of the dataset, rather than according to prior information like sectoral shares. The correlated variables are then converted into a set of orthogonal, linearly uncorrelated variables called principal components.

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[9] Various measures of inflation (including CPI, core CPI, and some measures of services inflation) and the unemployment rate were also considered but ultimately were left out of the final analysis.

Cyclical indicators need to be combined and weighted to produce an aggregate output gap estimate. We therefore transform the series into comparable units of measurement before deriving weights for these to produce an output gap estimate. The various cyclical indicators selected are standardised prior to estimation, i.e. they are expressed as a number of standard deviations from the mean of the series. For any given variable *x*, the standardised value of that variable ($\hat{x}$) is given by the expression:

$$\hat{x} = \frac{x - \bar{x}}{\sigma_x} \qquad (9)$$

where $\bar{x}$ denotes the sample mean of the series and σ_x denotes the standard deviation. In each case we need to specify the sample period to calculate an appropriate mean and standard deviation of each series. This may not correspond to the entire time series: for indicators with a relatively short time span it may not be appropriate to use the whole sample if the starting point is during a period of elevated or depressed economic activity, as this may introduce a cyclical bias in the long-term average. One criterion for the period used to calculate the "normal" level of an indicator is whether the series is symmetrically distributed over that period.[10] As such we assess the distribution for symmetry during the period over which an average is taken.

4.1.4 Testing Results
Turner *et al.* (2016) suggest that in addition to targeting low real-time revisions, other criteria used to judge potential output estimates/methods might also include the ability to explain inflation; applicability across many countries; and a plausibility or "smell" test. In this spirit, we perform stability tests like those favoured in McMorrow *et al.* (2015) and elsewhere, but we also extend our tests in two important ways. We examine the ability of our output gap estimates to explain inflation and we propose a means of assessing the complexity of estimation procedures. The latter is an attempt to account for the trade-off between obtaining more appropriate measures at the cost of these becoming more opaque and/or more prone to error. Greater difficulty in interrogating the methodologies applied and reproducing estimates are major drawbacks of approaches as they become more complex.

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[10] As the variables are standardised prior to inclusion in the Principal Components Analysis, it is important that the conversion of the series to a Standard Normal distribution does not distort the shape of the underlying empirical distribution to any great degree. Examining kernel density estimates of the included series, we opt to exclude three variables from the analysis: Recreation and Culture Inflation, Unemployment, and Domestic Services and Household Services Inflation.

4.2 Data

Recognising the challenges posed in Section III, we focus on Domestic GVA as the main aggregate of interest. As the economic output attributable to the foreign-owned multinational dominated sector still forms a large and important part of the economy, notwithstanding the weak relationship with revenues, we express our output gap estimates as a share of potential Domestic GVA plus actual GVA of multinational enterprises. This is consistent with the approach availed of in IMF (2015), which corresponds to the view that the foreign-owned multinational dominated sector is always operating at full potential with the gap between potential output and actual output driven primarily by domestic developments.[11]

We also explore the inclusion of a number of additional observable variables in a multivariate setting. This approach is intended to provide further information about the cyclical component of output. By complementing actual economic aggregates (e.g. real GDP or real Domestic GVA) with additional information that could help to determine unobservable estimates of the output gap, we can improve the chances that appropriate estimates are identified. The additional variables can be construed as additional signal variables in a multivariate setting, which are intended to augment the signal provided by our actual economic activity measure.

The additional observable variables we examine as complementary to our aggregate measures of economic activity include: a modified measure of the current account balance; the real effective exchange rate; house price growth; real credit growth; and real interest rates. The inclusion of financial variables (real interest rates, house price growth and real credit growth) is intended to capture the influence of the financial cycle and in particular the influence that developments such as asset price booms or credit expansions may have in determining cyclical developments and budgetary outcomes. For an alternative cyclical indicators approach, we investigate a battery of additional survey indicators that could serve to identify the cyclical position of the economy. Typically, these approaches focus on survey measures of spare capacity and recruitment difficulties, along with official data on variables that signal overheating through the price/wage channel (Pybus, 2011; Murray, 2014).

On the inflation front, we choose variables that might be more closely aligned with domestic price pressures. Previous research for Ireland has shown strong predictive power for components of domestic services inflation using short-run unemployment gaps as a proxy for domestic spare capacity (Bermingham *et al.*, 2012). In this spirit, we focus on components of services inflation that may be said

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[11] Rather than using GNI*, we use Domestic GVA. This choice is largely motivated by (i) the lack of inflation-adjusted data for the new GNI* measure; (ii) the fact that Domestic GVA provides a clearer separation of sectors dominated by foreign-owned multinational enterprises. The implied assumption is that there is no output gap for sectors dominated by foreign-owned multinational enterprises. This approach would be consistent with the view that the sector faces limited resource constraints over time, can draw on a wider labour pool, and has no capital or efficiency gap.

to represent the non-traded element of domestic inflation or the more sheltered sectors of the economy. We include annual price inflation for restaurants and hotels; recreation and culture; transport services; and private rents.

Table 2: Summary of Variables Assessed

	Unit	*Source*
Aggregate Macroeconomic Measure		
Domestic GVA	Log of level in €m (2015 prices)	CSO
Additional Signal Variables for Multivariate Filters		
Modified Current Account Balance[1]	% GNI*	CSO
House Prices	% change y/y	BIS
Private Sector Credit growth	% change y/y	CBI
Short-Term Real Interest Rates (1yr; CPI inflation)	%	Thomson Reuters
Real Effective Exchange Rate (CPI-based, 67 partners)	% change y/y	Bruegel
Variables for Cyclical Indicators Approach		
Exchange Rate: USD-EUR	$/€	Thomson Reuters
Restaurants and Hotels Inflation	% change y/y	CSO
Recreation and Culture Inflation	% change y/y	CSO
Transport Services Inflation	% change y/y	CSO
Domestic Services and Household Services Inflation	% change y/y	CSO
Private Rent Inflation	% change y/y	CSO
Construction Sector PMI	Index	Markit
Services Sector PMI	Index	Markit
Housing Completions	(4-Qtr moving sum) – (LR avg)[2]	CSO
New Vehicle Registrations	% change y/y	CSO
Unemployment Rate	% labour force	CSO

Notes: CBI = Central Bank of Ireland; CSO = Central Statistics Office; BIS = Bank for International Settlements;

[1] The modified current account balance is the Balance of Payments current account balance less the impact of re-domiciled PLCs, depreciation of intellectual property and leased aircraft, research and development imports, net purchases of intellectual property products, and investment into intellectual property assets and aircraft leasing.

[2] The long-run average used here is set as the annual average for the period 1975-2015 (which approximates as 25,000 units), excluding 2003-2009 to account for the bubble period.

V ESTIMATION RESULTS

Figure 1 summarises the results of the estimations. In each of the uni- and multivariate filters, we filter data on Domestic GVA from 1970-2016.[12] We do not use forecasts in the estimations shown, though forecasts could be used to alleviate end-point bias.[13] Output gap estimates are expressed as actual Domestic GVA as a share of potential Domestic GVA + actual GVA of sectors dominated by multinational enterprises. Of the variables we examine for inclusion in the multivariate approach, we find house prices and the adjusted current account balance to be significant at the 10 per cent level. All other variables are found to be insignificant. The other specifications explored in Borio *et al.* (2013), which include credit and real interest rate indicators for example, prove less useful given that the variables are all found to be insignificant at the 10 per cent level. In part, this may reflect data availability considerations and the approaches could yet prove useful in time.

Results provided by the HP filter with a lambda of 100 and the Kalman filter estimates (both with and without a drift term) suggest an output gap that is positive for most of the early 2000s, then turning negative after the financial crisis begins in 2009 and only turning consistently positive from either 2014 or 2015 onwards. An exception is the HP filter approach that uses a lambda smoothing parameter of 6.25 (consistent with a cycle of ten years). Results for this approach give a pattern largely different to the other three. In particular, it produces implausible negative estimates from 2002-2004, and an estimate that is more positive than any other method for 2011 despite the widespread downturn in the economy. In terms of magnitude, there is some notable variation between the methods applied. The HP filter with a lambda of 100 displays wider swings than other methods, with a peak of +7.2 per cent in 2007 and a trough of –4.8 per cent in 2012. By contrast, the estimates produced using the Kalman filter give estimates that are much shallower. The Kalman filter (without a drift term) gives estimates that are quite close to zero for the period 2009-2013, which is at odds with evidence of spare capacity in the economy during this period including the large increases in unemployment.

Looking at the multivariate filter estimates, both the results including house prices and the adjusted current account balance point to a positive output gap sustained over the 2000s. This positive output gap widens from 2003 as the housing bubble takes hold, with associated negative impacts on the current account balance

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[12] We extend the outturn Domestic GVA data available for 1995-2016 backwards by linking its growth rates to growth rates for real GNP, with which there tends to be a high correlation. We do this by estimating the typical relationship between the two variables econometrically over the period 1995-2016 based on the relationship (with variables in log-levels): $\Delta GNP_t = \alpha + \beta \Delta Domestic\_GVA_t + \varepsilon_t$.

[13] Though forecasts are not typically available for Domestic GVA, they are available for variables such as GNP, which estimates tend to be highly correlated with. This offers one exploitable solution when trying to alleviate end-point bias.

and rising house prices.[14] The subsequent reversal in the output gap estimates produced under both methods coincides with the collapse of the bubble in 2009. The estimates that control for house prices produce a much deeper negative output gap in the period 2010-2014 compared to the estimates that incorporate the adjusted current account balance. This might be expected, given the relatively greater deviation from long-run levels observed in house prices, compared to deviation observed for the current account balance. The former yields estimates that hit a trough at close to 5 per cent, while the latter bottom out at a negative output gap of 3 per cent. Both sets of estimates show a return to modest positive gaps in 2016 of between 0.2 per cent and 0.9 per cent.

Figure 1: Mid-Range Output Gap Estimates
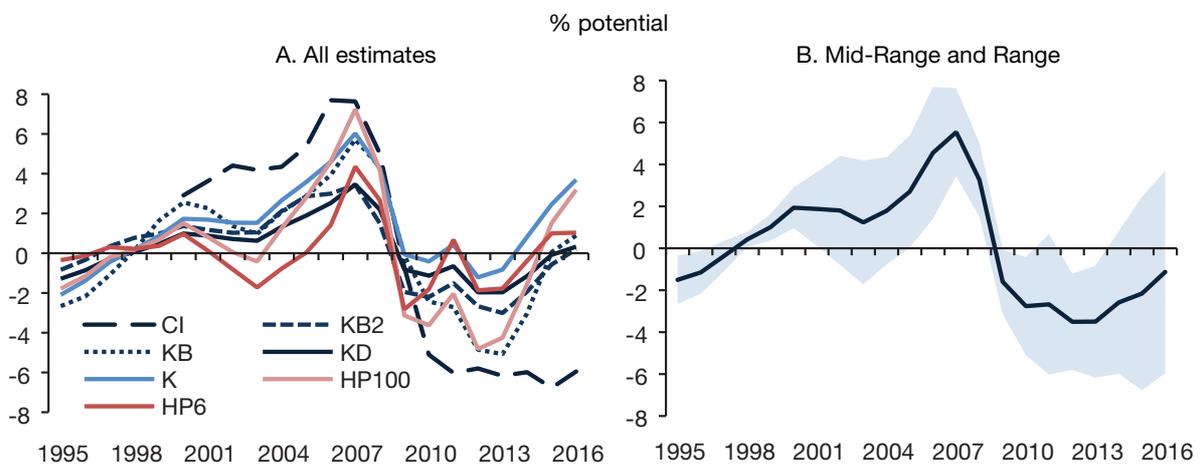
Sources: Author's calculations.
Note: Mid-Range estimates are computed as the series of averages of the maxima and minima of estimates produced under each method in each period. "HP6" refers to the HP filter Domestic GVA estimates (λ=6.25); "HP100" refers to same with different smoothing parameter (λ=100); "K" refers to the Kalman filter Domestic GVA estimates; "KD" refers to the Kalman filter of Domestic GVA with a drift term; "KB" refers to Kalman filter of Domestic GVA with drift term and house prices; "KB2" refers to the Kalman filter of Domestic GVA with a drift term and the adjusted current account balance. "CI" refers to the Cyclical Indicators estimates of the output gap; "Mid-Range" refers to the average of the maxima and minima of each of the preceding methods for each period; while "CAM" refers to the European Commission estimates of the Irish output gap using the Commonly Agreed Methodology.

In terms of the Cyclical Indicators approach, variables chosen for the construction of principal components estimates should reflect whether the series is symmetrically distributed over the full sample period. As in Bulmer (1979), we take skewness

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[14] Both Whelan (2013) and Honohan (2010) show evidence that the period 2002-2003 marked the onset of the bubble period leading to Ireland's subsequent crisis.

measures of between −½ and +½ for indicator distributions as being approximately symmetric. Excluding indicators whose distributions show skewness greater than 0.5 means we drop the following variables as indicators for consideration in the principal components analysis: Recreation and Culture Inflation; the Unemployment Rate; and Domestic Services and Household Services Inflation. The variables are standardised prior to the computation of the weights, with the mean and standard deviation based on the sample period Q1 1996 – Q4 2002. This period is chosen to reflect a period when (i) the property/credit bubble had not yet begun to cause severe distortions in data and when the effects of a delayed convergence were sufficiently borne out; (ii) when the economy could be said to have been broadly in equilibrium with a balanced current account and relatively low and stable inflation rates; and, of course, (iii) data availability considerations.

Taking the first principal component, we find that this produces a reasonable representation of how the output gap might be seen to have evolved over the past two decades. A modest positive output gap estimate of around 2 per cent is recorded in the early 2000s. We then see a gradual widening of the output gap in subsequent years, which accelerates in the 2005-2007 period, consistent with the peak of the property-credit bubble. As the bubble collapses, the output gap turns sharply negative, before hitting a trough in 2009-2010. Following a prolonged stagnation, a gradual recovery begins to take place after 2015, with an output gap of –3.8 per cent as of mid-2017. The persistent and deep negative output gap is far more negative than that shown by other measures. In part, this reflects the moderating influence that falling unemployment rates might have (excluded from the estimation process), while sluggish housing completions are likely to have a large negative bearing on the estimates.

It is important to note that Principal Component Analysis is a high-dimensional approach, requiring large data sample for asymptotic properties to hold. With a sample of less than 15 variables and 25 time points, this could present a challenge for our estimates.

5.1 A Suite of Models Approach

Following the suite of models approach, we examine a set of estimates that combines the information from alternative methods we propose in this section (i.e. the univariate and multivariate filters, and the Cyclical Indicators approach). There are a number of approaches that could be adopted in terms of model averaging (e.g. simple arithmetic average; information criterion model averaging; Bayesian model averaging, etc.). Here, we employ a relatively simple approach, which is to take the mid-point of the full-range of estimates from the models we have identified. The "Mid-Range" estimates are computed as the year-wise averages of the maxima and minima of estimates produced under each method. For example, in 2010, the Mid-Range estimate is simply the average of the maximum and minimum estimate produced for that year based on all of the methods employed. In the second panel

of Figure 1, we show the Mid-Range estimates overlaid on the full range of model estimates.

The approach is useful for several reasons: (i) it is not unduly influenced by methods that we use several times, despite these being functionally quite similar (e.g. various modifications of the Kalman filter); (ii) it is simple to calculate; and (iii) it presents an intuitive interpretation in the context of a range of estimates. It has a number of drawbacks, most notably that it does not incorporate model selection criteria that may be relevant (e.g. selection on the basis of stability, ability to explain inflation, etc.). However, as a starting point, it serves as a useful basis for testing.

Of course, a mid-range will not always be an optimal basis for arriving at central estimates. As the results will depend on the most extreme estimates from the set of models, it can mean that estimates may be biased by outliers. Alternative approaches to deriving central estimates of the output gap other than a mid-range include the mean or median estimate of the suite of models. These approaches have the advantage of incorporating all estimates in the calculation of the central value, hence limiting the extent to which extreme values drive the result. However, these approaches also have their drawbacks. Many of the models have a similar structure (e.g. the multivariate Kalman filters are broadly similar in structure, save for the inclusion of different signal variables). This similarity could lead us to use central mean or median estimates that place too great a weight on a number of models that, rather than adding much new information, are simply greater in number.

Ultimately, the design of central estimates to the set of models that we have produced presents a number of difficult trade-offs. There is no perfect answer as to how to summarise these in a single set of estimates. We favour the mid-range on the grounds that it tends to capture the central tendency of the different models we consider for the period considered, without overweighting any particular model form. However, this could change especially if outliers were to become more of a problem. In practice, the full suite of models should be considered individually in order to better inform policy, though producing summary estimates is often a necessity in public policy.[15] Another useful approach – not considered here – is to formally incorporate the trade-offs into a selection algorithm as in Cuerpo *et al.* (2018).[16]

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[15] In describing policymakers' strong preference for single, summary estimates, Manski (2013) recounts the (possibly apocryphal) story of an economist's attempt to describe his forecast uncertainty to President Lyndon B. Johnson. The economist presents his forecast as a likely range of values. Johnson responds by noting "Ranges are for cattle. Give me a number".

[16] The authors consider three necessary conditions for selecting output gap estimates: economic soundness, statistical goodness, and transparency.

VI TESTING THE RESULTS

Given the output gaps we construct, we next examine a number of tests to discern the quality of these estimates on the basis of some features which may be desirable or undesirable: (1) stability; (2) how informative real-time estimates are; (3) the ability to explain inflation; and (4) complexity.

6.1 Test 1: Stability of Output Gap Estimates

Stability tests involve an examination of how stable the estimates are over time as new vintages of data/forecasts are produced. The idea is to compare repeated iterations of the same methodology over time. We repeat this test for each of the methods explored and using real-time data where relevant (e.g. for Domestic GVA and the for the current account balance). Therefore, the revisions are either the result of (i) revisions to real-time data or (ii) the re-estimation of the model for each period.

As the Domestic GVA series was first published in 2013, and hence real-time estimates only started to be collected from that point onwards, we create a pseudo real-time series. We construct these estimates by exploiting the close historical association between real GNP and Domestic GVA.[17]

To test stability, we examine three measures: (i) the Mean Absolute Revision (MAR); (ii) the max revision; and (iii) the number of sign changes observed. We calculate revisions in two ways: first in terms of year-to-year revisions of estimates (e.g. the revision to the 2000 output gap estimate as estimated in 2006 relative to 2005, then in 2007 relative to 2006, and so on). In this case, the MAR is computed as:

$$MAR_t = \frac{1}{n} \sum_{i=1}^{n} |x_{t,i} - x_{t,i-1}| \tag{10}$$

where x_i is the ith output gap estimate for a given year t. A summary measure is then obtained by averaging across all of the MAR estimates for each t year.

Second, we can calculate revisions in terms of the final estimate less the initial estimate (e.g. the 2000 output gap estimate as finally estimated in 2016 minus its initial estimate using data up to 2000). The MAR for a given year's output gap estimate is therefore the average of all absolute revisions for each year where the latter are calculated as:

$$Absolute\ Revisions = |x_{t,final} - x_{t,initial}| \tag{11}$$

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[17] A simple regression of Domestic GVA on a constant and real GNP (traditionally a better measure for the domestic economy in Ireland) – and both in log-differences – can be shown to explain historical variation quite well. We assume that this relationship (estimated over the full sample period 1995-2016) holds to create a fitted real-time series of Domestic GVA based on a real-time series for real GNP.

"Sign changes" refer to a simple count of the number of changes of the sign on the output gap from one vintage to the next (or from the initial to final estimates), i.e. from positive to negative or vice versa.

Table 3: Revisions to Output Gap Estimates
Full Sample (1999-2015)

	HP6	HP100	K	KD	KB	KB2	CI	Mid-Range	CAM
Year-to-Year Revisions									
MAR	0.5	0.8	0.6	0.3	0.4	0.5	1.2	0.5	0.7
Max Revision	5.5	6.9	3.0	1.8	3.2	2.3	7.1	3.4	4.2
Sign Changes	5	9	3	2	2	6	14	1	17
Initial-Final Revision									
MAR	2.7	4.6	2.9	1.1	2.5	1.1	4.4	1.7	2.3
Max Revision	6.5	8.6	6.4	2.1	5.7	2.2	8.9	3.4	5.4
Sign Changes	5	4	4	0	2	0	4	1	5

Sources: Author's calculations.
Note: "MAR" refers to the Mean Absolute Revision from one vintage to the next over all available estimates. "Sign changes" refer to a simple count of the number of changes of the sign on the output gap from one year to the next (i.e. from positive to negative or vice versa). "HP6" refers to the HP filter Domestic GVA estimates (λ=6.25); "HP100" refers to same with different smoothing parameter (λ=100); "K" refers to the Kalman filter Domestic GVA estimates; "KD" refers to the Kalman filter of Domestic GVA with a drift term; "KB" refers to Kalman filter of Domestic GVA with drift term and house prices; "KB2" refers to the Kalman filter of Domestic GVA with a drift term and the adjusted current account balance. "CI" refers to the Cyclical Indicators estimates of the output gap; "Mid-Range" refers to the average of the maxima and minima of each of the preceding methods for each period; while "CAM" refers to the European Commission estimates of the Irish output gap using the Commonly Agreed Methodology.

Table 3 shows the summary results of our stability tests for each output gap methodology as would be estimated in real time. We also include, for reasons of comparison, the real-time estimates produced under the EU Commonly Agreed Methodology for Ireland.[18] The results suggest that the estimates produced under the Kalman filter with drift (KD) and the Kalman filter with drift and the adjusted current account balance (KB2) have the lowest revisions. In terms of year-to-year revisions, the KD has a mean absolute revision of 0.3 percentage points, while the same measure for the Kalman filter with drift and house prices (KB) is 0.4 percentage points. For the KB2 method, the revisions are marginally higher at 0.5

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[18] Note that since we use the real-time output gap estimates published by the European Commission (CIRCABC Database) under the EU Commonly Agreed Methodology, these reflect methodological changes that took place over the same period of time.

percentage points. If we consider the revisions in terms of initial minus final estimates, we see that the KD and KB2 methodologies show the lowest revisions on average, at 1.1 percentage points. By comparison, this is less than half of that observed for the CAM. The largest revisions tend to be observed for the HP filter with a lambda of 100 and for the Cyclical Indicators approach. However, the latter partly reflects the shorter time period over which the estimates are able to arrive at a stable set of results (i.e. the first estimation window is for Q1 1996 to Q1 2003).[19]

A striking number of sign changes are evident for some of the methods. Most notably, the CAM shows as many as 17 sign changes on a year-to-year basis. Closest to that is the Cyclical Indicators approach (with 14 sign changes) and the HP filter with a lambda of 100 (nine sign changes). Looking at just the initial-to-final revisions, we see that the CAM also displays a large number of sign changes (5 over the full sample period). This makes it joint highest with the HP filter with a lambda of 6.25 ("HP6"). By comparison, the Kalman filter estimates show very few – if any – sign changes over time, both in terms of the year-to-year and initial-to-final revisions. Looking at the Mid-Range estimates, this approach produces results that are, as may be expected, relatively stable. In all cases, its revisions are typically only higher than those produced for two or three other methods; they are lower than those produced under the CAM; and only one sign is evident over time in terms of both year-to-year and initial-to-final revisions.
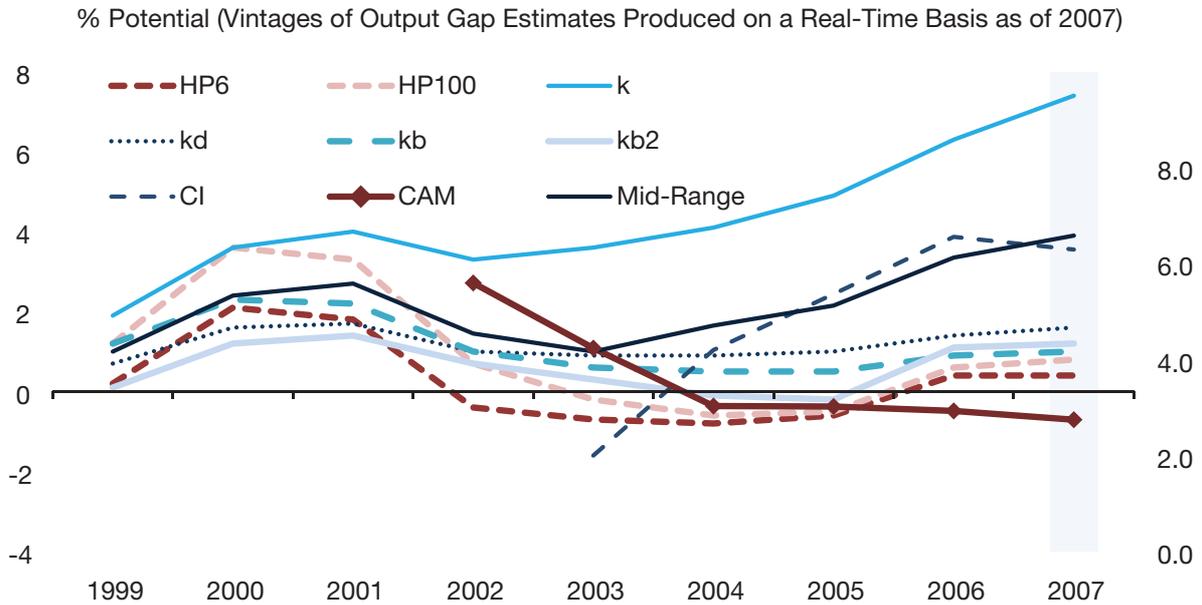
6.2 Test 2: How Informative are Real-Time Estimates

Another test related to the real-time performance of various methods is their informational value at key turning points. An obvious vintage to examine in this regard is the first estimates produced as of 2007 (that is the estimate produced immediately after official 2007 National Accounts data become available in June 2008). To be of use to those assessing economic policy, initial output gap estimates should give a fairly clear sense of possible demand excesses or shortfalls. At the very least, they should communicate a sign of the output gap that is in keeping with concurrent economic developments.

We examine the first 2007 outturn vintage of output gap estimates produced under each methodology. Aside from the CAM-based estimate for 2007 (-0.7 per cent), all of the methods indicate a positive output gap (Figure 2). Given that this was the peak of the credit/housing bubble, a large positive output gap would be expected. By comparison, the Mid-Range estimate is 3.9 per cent and lies mid-way between the univariate Kalman filter estimate of 7.4 per cent and the HP filter estimate with $\lambda = 6.25$. Though positive, the HP filter-based estimates have relatively small magnitudes considering the scale of the demand excess that might have been expected for 2007, as do the Kalman filter estimates that control for house prices (1 per cent) and the adjusted current account balance (1.2 per cent).

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[19] The estimates for a smaller more recent sample period (2010 to 2015) reveal smaller revisions relative to the CAM and other methods, both on a year-to-year and initial-to-final basis.

Figure 2: Comparison of the 2007 Vintage of Estimates

% Potential (Vintages of Output Gap Estimates Produced on a Real-Time Basis as of 2007)

Sources: Author's calculations.
Note: "HP6" refers to the HP filter Domestic GVA estimates (λ=6.25); "HP100" refers to same with different smoothing parameter (λ=100); "K" refers to the Kalman filter Domestic GVA estimates; "KD" refers to the Kalman filter of Domestic GVA with a drift term; "KB" refers to Kalman filter of Domestic GVA with drift term and house prices; "KB2" refers to the Kalman filter of Domestic GVA with a drift term and the adjusted current account balance. "CI" refers to the Cyclical Indicators estimates of the output gap; "Mid-Range" refers to the average of the maxima and minima of each of the preceding methods for each period; while "CAM" refers to the European Commission estimates of the Irish output gap using the Commonly Agreed Methodology.

6.3 Test 3: Output Gap Estimates Ability to Explain Inflation

Another way to examine the plausibility of the output gap estimates produced under different methods is to test their ability to explain inflation. This forms the basis of our third set of tests.[20]

To test our output gap estimates ability to explain inflation in the current year t (π_t), we first estimate a simple Phillips curve equation:

$$\pi_t = \beta_1 \pi_{t-1} + \beta_2 (Output\ gap) \tag{12}$$

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[20] An important consideration in such tests is whether the models already explicitly or tacitly include inflation indicators so that they are forcing output gaps to explain inflation. Such approaches could introduce large biases and may represent an overly restrictive way of incorporating economic information into statistical methods (Borio *et al.*, 2013). With the exception of the cyclical indicators approach, the estimates of domestic output gaps we produce do not incorporate inflation measures. However, the estimates that incorporate house prices will, of course, have some endogeneity to general price inflation.

before, second, exploring a more complex Phillips curve approach that incorporates inflation expectations and inflation targeting by a central bank:

$$\pi_t = \beta_1 \pi_{t+1}^e + \beta_2 (Output\ gap) \tag{13}$$

where inflation expectations for the next year (π_{t+1}^e) are given by:

$$\pi_{t+1}^e = \beta_3 \pi_t + (1 - \beta_3)\pi^{target} \tag{14}$$

with β_3 assumed to lie between zero and one, and the inflation target is given by π^{target}, which we assume to be 2 per cent (consistent with the ECB's mandate). This implies that individuals expect next period's inflation to be a weighted average of officially targeted inflation and past inflation.

The results incorporating inflation-targeting and inflation expectations (Table 4) show that our models perform broadly as well as the CAM. This is also true when using wage inflation, though results in a simple Phillips curve using CPI do not have as strong an explanatory power when compared to the CAM.[21]

6.4 Test 4: Complexity of the Estimation Process Involved

Testing the complexity of an estimation process is also desirable. It can have implications for how informative estimates are in terms of their drivers. Complexity can also be a useful predictor of the likelihood of defects occurring. However, measuring the complexity of estimation code is not a straightforward task. We could examine the length of time it takes to undertake the procedure but this can differ across iterations, with variations in processing power, human error, and differences in user experience and knowledge factors that would need to be controlled for. Also, some methods may require several programming tools to run and so the actual run time or length of code may be less well-defined. In the field of computer science, one useful approach to testing the complexity of an algorithm involves examining the number of statistical operations required. This is relatively easy for us to investigate, given that we code all of the models in the same software package so that the operations employed are comparable.[22] We count statistical operations as any operational commands used (e.g. sample selection, arithmetic, comparisons, accessing array's elements, assignment, etc.).

Table 5 summarises the complexity of the estimation methods. In terms of input series the CAM requires the most (14 inputs) as compared to 11 for the cyclical

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[21] Results for the simple Phillips curve setting are available on request.

[22] All of the models are coded in EViews 9.5, with the exception of two sections of the CAM code, which depend crucially on bespoke user interfaces designed to be used within Microsoft Excel. Those sections that we were not able to reproduce in EViews 9.5 are the parts that are designed to: (i) obtain estimates of Trend Factor Productivity by filtering the Solow Residual, and (ii) obtain estimates of the NAWRU based on a new Keynesian Phillips curve approach (Havik *et al.*, 2014).

Table 4: Output Gaps and Inflation (Phillips Curve Approach Incorporating Inflation Expectations and Inflation Targeting), 1990-2016

	HP6	HP100	K	KD	KB	KB2	CI	Mid-Range	CAM
Dependent variable: CPI inflation									
π^{exp}_{t+1}	1.20***	1.21***	1.23***	1.22***	1.23***	1.21***	1.22***	1.20***	1.15***
	(0.07)	(0.07)	(0.09)	(0.08)	(0.08)	(0.07)	(0.12)	(0.07)	(0.07)
Output Gap	0.38***	0.20***	0.09	0.33**	0.17**	0.32***	0.13**	0.25***	0.25***
	(0.11)	(0.06)	(0.08)	(0.12)	(0.06)	(0.09)	(0.05)	(0.07)	(0.07)
Observations	26	26	26	26	26	26	16	26	26
R-squared	0.85	0.84	0.78	0.83	0.83	0.85	0.85	0.86	0.86
Dependent Variable: Core CPI inflation									
π^{exp}_{t+1}	1.19***	1.20***	1.23***	1.22***	1.21***	1.20***	1.19***	1.19***	1.14***
	(0.06)	(0.07)	(0.07)	(0.06)	(0.06)	(0.06)	(0.10)	(0.06)	(0.06)
Output Gap	0.32***	0.17***	0.05	0.25**	0.14**	0.25***	0.11**	0.20***	0.22***
	(0.10)	(0.05)	(0.07)	(0.10)	(0.05)	(0.08)	(0.05)	(0.06)	(0.06)
Observations	26	26	26	26	26	26	16	26	26
R-squared	0.89	0.89	0.85	0.87	0.88	0.89	0.89	0.89	0.90

Sources: CSO; author's calculations.

Notes: Robust standard errors in parentheses (*** $p<0.01$; ** $p<0.05$; * $p<0.1$). "HP6" refers to the HP filter Domestic GVA estimates ($\lambda=6.25$); "HP100" refers to same with different smoothing parameter ($\lambda=100$); "K" refers to the Kalman filter Domestic GVA estimates; "KD" refers to the Kalman filter of Domestic GVA with a drift term; "KB" refers to Kalman filter of Domestic GVA with drift term and house prices; "KB2" refers to the Kalman filter of Domestic GVA with a drift term and the adjusted current account balance. "CI" refers to the Cyclical Indicators estimates of the output gap; "Mid-Range" refers to the average of the maxima and minima of each of the preceding methods for each period; while "CAM" refers to the European Commission estimates of the Irish output gap using the Commonly Agreed Methodology.

indicators approach and less than three inputs for all other approaches. In terms of statistical operations, the CAM far exceeds the complexity of any other method with over 160 operations involved and likely closer to 200 operations if parts of the estimation process that are conducted in other software were to be included (i.e. macro-enabled Microsoft Excel spreadsheets). This compares to between ten and 34 operations being required for all of the other methods. The number of operations involved in CAM estimation by comparison to other methods lends itself to greater risks of defects occurring. This risk is aggravated by the fact that changes to the code are frequent and, although available to general users through code provided on the European Commission's Circa website, can often be difficult to ascertain in a timely manner.

Table 5: Complexity of Estimation Methods

Method	*Number of Input Series*	*Number of Statistical Operations Involved*
HP Filter	2	10
Kalman Filter (KF)	2	28
KF with drift	2	31
KF with House Prices	3	34
KF with Current Account	3	34
Cyclical Indicators	11	24
CAM *	14	160+

Source: Author's calculations.
Notes: Statistical operations refer to all operational commands (arithmetic, comparisons, accessing array's elements, assignment, etc.). * The production of CAM-based estimates involves in excess of 160 operations when run through EViews for Ireland (excluding various "if" statements related to country selection). The total operations involved are likely closer to 200 given that we did not replicate two parts of the code: (i) that related to the NAWRU estimation procedure and (ii) that related to detrending of the Solow Residual.

VII CONCLUSIONS

This paper attempts to identify plausible estimates of Ireland's output gap that are relevant for fiscal policy. In particular, we seek to identify alternative approaches to the EU commonly agreed (production function) methodology. A number of challenges face us, particularly Ireland's small, open nature and the presence of large foreign-owned multinational enterprises. Recognising these challenges, we prioritise measures that focus on domestic activity, an approach warranted given its relatively more tax-rich nature. In addition, we use a suite of models approach, thus availing of a range of alternative estimation techniques rather than relying on any single approach.

Examining and testing methods based on univariate and multivariate statistical filters and principal components analysis, we find that the results produce more plausible estimates than the Commonly Agreed Methodology (CAM) estimates. The alternative estimates also tend to be as stable as CAM-based estimates and are far less complex to estimate. The alternative estimates have a similar explanatory power when incorporating price expectations and inflation-targeting or when considering wage inflation instead of price inflation, although their ability to explain price inflation in a standard Phillips curve setting is weaker than that for the CAM.

Yet we do not see these alternative estimates as a panacea to identifying cyclical developments and imbalances in the economy. Every cycle is different and keeping analysis simple and with a clear narrative is problematic in a complex world. Designing a "least bad" solution among a host of mediocre choices might be the only realistic goal for the problem of estimating potential output (Blagrave *et al.*, 2015). In particular, it would be worthwhile developing alternative estimates of the output gap and potential output in the context of a full (semi-) structural model approach. This would ensure a more robust foundation for any estimates produced. Moreover, the concept of potential output might not correspond well with the concept of "sustainable" output. Estimates like those produced here need to be supplemented with a careful scrutiny of economic imbalances to discern whether developments could lead to subsequent painful corrections in the economy. Future research could expand on the "modular approach" proposed by the Irish Fiscal Advisory Council (2015), which examines a range of economic indicators for signs of economic imbalances. Modules may focus on areas such as the labour market; housing and investment; credit; and external balances.

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